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SOME RESULTS IN NOISE FILTERING BY USE OF LOCAL STATISTICS

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In a recent paper [1], J. S. Lee proposed a method of using local statistics in noise filtering. The basic assumption is that the ensemble mean and variance of a pixel is equal to the local mean and variance of all pixels within a fixed range surrounding it. So the gray level of any point of the original image can be estimated by the local statistics and the noisy gray level of that point. This implies that the image is locally stationary and ergodic. Though this assumption is not always exact, it has been shown to be quite feasible by some experimental results. We have implemented this method (for simplicity we'll call it L-method hereafter) in our minicomputer (PDP - 11/45).

1. Determination of Variance of Noise

In order to calculate the local statistics of the original image the variance of the additive noise σ_n^2 must be known. Sometimes only the noisy image ia available. In the case where the image is composed of objects and a nearly uniform background, σ_n^2 can be calculated from that window which has the least mean value. As the mean of noise is zero, the window with the least mean must be the uniform background and its variance must be σ_n^2. In the experimental study an uniform noise (σ_n^2=300) was added.

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σ_n^2 calculated by above mentioned method is 270. This value was used in processing the noisy image. The result is shown in Fig. 1.

2. Relation to Other Filtering Methods

First let us calculate the mean square error of the L-method. Assume that the signal x and noise n are independent Gaussian random variables, n has zero mean, and z is the measured gray level. Then we have

$$z_{ij} = x_{ij} + n$$

The conditional probability density p(x/z) must be Gaussian with [2] mean $[x]$ and variance $[\sigma^2]$:

$$[x] = \bar{x} + \frac{\sigma_x^2}{\sigma_z^2}(z_{ij} - \bar{z}) \qquad (1)$$

$$[\sigma^2] = \frac{\sigma_x^2 \sigma_n^2}{\sigma_x^2 + \sigma_n^2} \qquad (2)$$

where $\bar{x}$ is the mean of x_{ij} and $\bar{z} = \bar{x}$, $\sigma_z^2 = \sigma_x^2 + \sigma_n^2$. The MAP estimation $\hat{x}$ of x_{ij} is equal to $[x]$, this is the formula used to estimate x_{ij} in the L-method. The mean square error of this estimator $E(\|x-\hat{x}\|/z)$ is simply the variance of p(x/z),

$$\overline{e_l^2} = \frac{\sigma_x^2 \sigma_n^2}{\sigma_x^2 + \sigma_n^2} = \frac{\sigma_x^2}{1+b} = \frac{b}{1+b}\sigma_n^2$$

where $b = \frac{\sigma_x^2}{\sigma_n^2}$ is the signal-to-noise ratio.

A. Relation to Local Averaging

In the local averaging method we replace every x_{ij} by $\bar{x}$. So the average square error $\overline{e_a^2}$ must be

$$\overline{e_a^2} = E(x-\hat{x})^2 = E(x-\bar{x})^2 = \sigma_x^2 = b\sigma_n^2$$

$$\frac{\overline{e_l^2}}{\overline{e_a^2}} = \frac{1}{1+b}$$

When $b \to 0$, $\overline{e_a^2} \doteq \overline{e_l^2}$. $b \to 0$ means $\sigma_x^2 \to 0$ i.e. the image is nearly uniform hence $\bar{x}$ is the optimal estimator of x, When $\sigma_x^2 \neq 0$, L-method

is superior to local averaging.

B. Relation to the Kalman Filter

In Kalman filter we estimate x_{ij} by recursive estimation. For the points that do not have neighborhood points information the filtering formula becomes (See [3] Chapter 7 formula (142,143). Also [4] formula (7.46)) $\hat{x}_{ij} = \bar{x} + k(z-\bar{x})$

where $$K = \frac{K_{ff}(0,0)}{K_{ff}(0,0) + \sigma_n^2} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_n^2}$$

which is the same formula used in the L-method. (See equation(1)). We can conclude that L-method is nothing more than a special case of the Kalman filtering and its effect is inferior to Kalman filter in the global operation. However, its computing algorithm is simpler and can be directly implemented for real-time processing where a parellel processor is used.

The effect of filtering is dependent upon the characteristics of the images under processing. If the image is nearly uniform or the gray level of one pixel is independent of other pixels, then local mean is the best estimator. If the gray level of one pixel is related to other pixels but the relationship is unknown, the L-method is the best estimator. If this relationship can be assumed as a suitable model, then Kalman filter is the best estimator. If the image has more complex characteristics, we must use some non-linear methods. Here "best" means the least mean square error.

3. An Alternative Approach in the Case of Multiplicative Noise

Under multiplicative noise the degraded pixel can be represented by $$z_{ij} = x_{ij}\, u_{ij} \qquad (3)$$
where u_{ij} is noise. In Ref. 1, this problem is treated by a linear relation between z, x and u:

$$z'_{ij} = Ax_{ij} + Bu_{ij} + C$$

A, B and C are determined such that the mean square error between z_{ij} and z'_{ij} is minimized. The resulting filtering formula is $\hat{x}_{ij} = \bar{x} + K_{ij}(z_{ij} - \bar{u}\bar{x})$ which is $\bar{x}$ plus some correction term. Intuitively for the multiplicative noise this formula is not reasonable as in the additive noise case. Therefore we propose another approach. Intuitively the reasonable form of $\hat{x}$ may be

$$\hat{x}_{ij} = Kz^{c}_{ij}$$ Now we take log of (3),

and assume $\widehat{\ln x}$ be a linear function of ln z i.e. $\widehat{\ln x} = c \ln z + k$ (or $\hat{x} = e^{k}z^{c}$)*. Here c, k are determined such that the mean square error between ln x and $\widehat{\ln x}$, $E(\ln x - \widehat{\ln x})^2$ is minimized.

$$\therefore \quad C = \frac{E\ln^2 x - (E\ln x)^2}{E\ln^2 z - (E\ln z)^2} = \frac{E\ln^2 x - (E\ln x)^2}{[E\ln^2 x - (E\ln x)^2] + [E\ln^2 u - (E\ln u)^2]} \tag{4}$$

$$k = E\ln x - C\,E\ln z = E\ln x - C(E\ln x + E\ln u) \tag{5}$$

The problem is to calculate $E\ln^2 x$, Eln x...etc. If the statistics of noise is known, i.e. we know $\bar{u}$ and σ_u^2, then the $\bar{x}$ and σ_x^2 can be determined from the following equations:

$$\bar{x} = \frac{\bar{z}}{\bar{u}}$$

$$\sigma_x^2 = \frac{\sigma_z^2 - \bar{x}^2\sigma_u^2}{\sigma_u^2 + \bar{u}^2}$$ (see Appendix)

Here $\bar{z}$ and σ_z^2 are the local mean and the variance of the noisy image.

With a reasonable assumption of the probability density of x, $E\ln^2 x$ and Eln x can be calculated from

$$E\ln^2 x = \int \ln^2 x \; p(x)\,dx$$
$$E\ln x = \int \ln x \; p(x)\,dx$$

*It is the same principle as Homomorphic filtering. Here the processing is in the spatial domain using the local statistics.

Here we simply list the resulting filtering formula. And for detail, please see the appendix.

$$S = E\ln x = \frac{F}{2} - H - 1.5$$

$$T = E\ln^2 x = \frac{G}{2} - \frac{3}{2}F + H(3 - \ln\bar{x}) + 3.5$$

$$C = 1 - V/(T - S^2 + V)$$

$$k = S - C(S - \bar{u})$$

$$\widehat{\ln x_{ij}} = C\ln z_{ij} + k$$

$$\hat{x}_{ij} = e^{\widehat{\ln x_{ij}}}$$

where

$$H = \left(\frac{\bar{x}}{2.5\sigma_x}\right)^2 \ln\bar{x} \; ; \qquad D = \bar{x} - 2.5\sigma_x$$

$$F = \left(\frac{D}{2.5\sigma_x}\right)^2 \ln D + \left(\frac{E}{2.5\sigma_x}\right)^2 \ln E \; ; \qquad E = \bar{x} + 2.5\sigma_x$$

$$G = \left(\frac{D\ln D}{2.5\sigma_x}\right)^2 + \left(\frac{E\ln E}{2.5\sigma_x}\right)^2 \; ; \qquad V = E\ln^2 u - (E\ln u)^2$$

Experimental results:

The multiplicative noise is uniform distritution from 0.2 - 1. After processing, the result is shown in Fig. 2A. For the case of combined multiplicative and additive noise, the result of processing is shown in Fig. 3. For comparison, with the same noise, the result of using the L-method is shown in Fig. 2B.

References

(1) J. S. Lee, "Digital image enhancement and noise filtering by use of local statistics," IEEE Trans. PAMI-2, no. 2, 1980. p.165

(2) B. D. O. Anderson and J. B. Moore, "Optimal Filtering", Prentice-Hall, 1979.

(3) A. Rosenfeld and A. C. Kak, "Digital Picture Processing", Academic Press, 1976.

(4) M. Schwartz and L. Shaw, "Signal Processing", McGraw-Hill, 1975.

Appendix

1, Derivation of σ_x^2

$$\sigma_z^2 = E(z-\bar z)^2 = Ez^2-(Ez)^2, \quad \sigma_x^2 = Ex^2-(Ex)^2, \quad \sigma_u^2 = Eu^2-(Eu)^2$$

$$\sigma_z^2 = E(xu-\bar x\bar u)^2 = E(x^2u^2-2\bar x\bar u\, xu+\bar x^2\bar u^2) = Ex^2\cdot Eu^2-\bar x^2\bar u^2$$

$$\sigma_x^2\sigma_u^2 = (Ex^2-\bar x^2)(Eu^2-\bar u^2) = Ex^2\cdot Eu^2-\bar x^2\bar u^2-\bar u^2(Ex^2-\bar x^2)-\bar x^2(Eu^2-\bar u^2)$$

$$= \sigma_z^2-\bar u^2\sigma_x^2-\bar x^2\sigma_u^2$$

$$\therefore \quad \sigma_x^2 = \frac{\sigma_z^2-\bar x^2\sigma_u^2}{\sigma_u^2+\bar u}$$

2. Calculation of Elnx, Eln^{2}x

In practice the distribution is usually assumed to be Gaussian. For simplicity we consider a triangular distribution (Fig. 4) as an approximation to the Gaussian distribution. The distribution function of x:

$$p(x)=\begin{cases}\dfrac{x-(\bar x-2.5\sigma_x)}{(2.5\sigma_x)^2}, & x=(\bar x-2.5\sigma_x)\rightarrow\bar x\\[2ex] \dfrac{\bar x+2.5\sigma_x-x}{(2.5\sigma_x)^2}, & x=\bar x\rightarrow(x+2.5\sigma_x)\end{cases}$$

$$\therefore\quad E\ln x = \int_{\bar x-2.5\sigma_x}^{\bar x}\frac{x-(\bar x-2.5\sigma_x)}{(2.5\sigma_x)^2}\ln x\,dx+\int_{\bar x}^{\bar x+2.5\sigma_x}\frac{\bar x+2.5\sigma_x-x}{(2.5\sigma_x)^2}\ln x\,dx$$

$$=\frac{1}{(2.5\sigma_x)^2}\left[\int_{\bar x-2.5\sigma_x}^{\bar x}x\ln x\,dx-(\bar x-2.5\sigma_x)\int_{\bar x-2.5\sigma_x}^{\bar x}\ln x\,dx+(\bar x+2.5\sigma_x)\int_{\bar x}^{\bar x+2.5\sigma_x}\ln x\,dx-\int_{\bar x}^{\bar x+2.5\sigma_x}x\ln x\,dx\right]$$

$$E\ln^2 x = \int_{\bar{x}-2.5\sigma_x}^{\bar{x}} \ln^2 x \, \frac{x-(\bar{x}-2.5\sigma_x)}{(2.5\sigma_x)^2}\, dx + \int_{\bar{x}}^{\bar{x}+2.5\sigma_x} \ln^2 x \, \frac{\bar{x}+2.5\sigma_x - x}{(2.5\sigma_x)^2}\, dx$$

$$= \frac{1}{(2.5\sigma_x)^2}\Big[\int_{\bar{x}-2.5\sigma_x}^{\bar{x}} x\ln^2 x\, dx - (\bar{x}-2.5\sigma_x)\int_{\bar{x}-2.5\sigma_x}^{\bar{x}} \ln^2 x\, dx$$

$$+ (\bar{x}+2.5\sigma_x)\int_{\bar{x}}^{\bar{x}+2.5\sigma_x} \ln^2 x\, dx - \int_{\bar{x}}^{\bar{x}+2.5\sigma_x} x\ln^2 x\, dx\Big]$$

By using the formulae,

$$\int \ln x\, dx = x\ln x - x + C \quad . \quad \int x\ln x\, dx = x^2\Big[\frac{\ln x}{2} - \frac{1}{4}\Big] + C$$

$$\int x\ln^2 x\, dx = \frac{x^2}{2}\ln^2 x - x^2\Big[\frac{\ln x}{2} - \frac{1}{4}\Big], \quad \int \ln^2 x\, dx = x\ln^2 x - 2x\ln x + 2x$$

the results can be easily derived.

3. Calculation of $E\ln u$, $E\ln^2 u$

When the noise u is uniformly distributed from a to b, then

$$E\ln u = \frac{1}{b-a}\int_a^b \ln u\, du = \frac{b\ln b - a\ln a}{b-a} - 1$$

$$E\ln^2 u = \frac{1}{b-a}\int_a^b \ln^2 u\, du = \frac{b\ln^2 b - a\ln^2 a - 2b\ln b + 2a\ln a}{b-a} + 2.$$

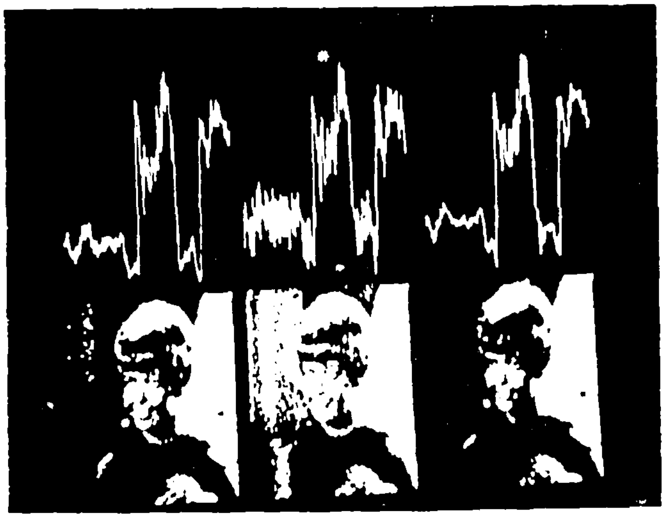

Fig. 1
Filtering of noisy image with additive noise, $\sigma_n^2 = 300$, of uniform distribution (-30, 30). The upper figure is the intensity profile along a scan line of the image in the lower figure. Original image in the left, noisy image in the middle, filtered image in the right.

Image is from USC data base, #USC-02568-15.

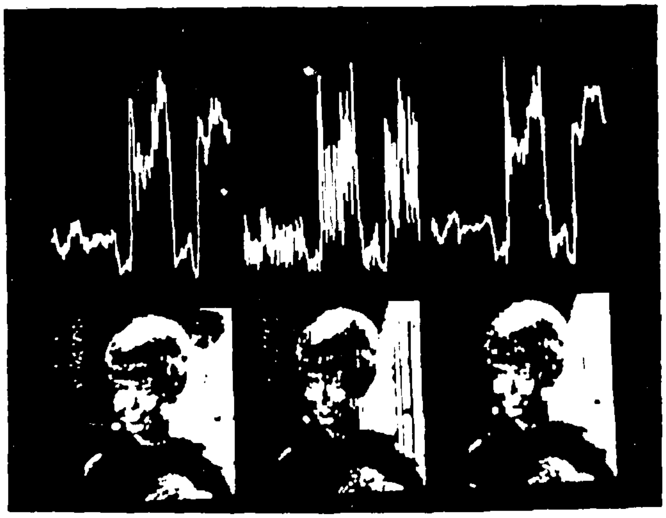

Fig. 2A
Filtering of noisy image with multiplicative noise of uniform distribution (0.2-1.) by using our method.

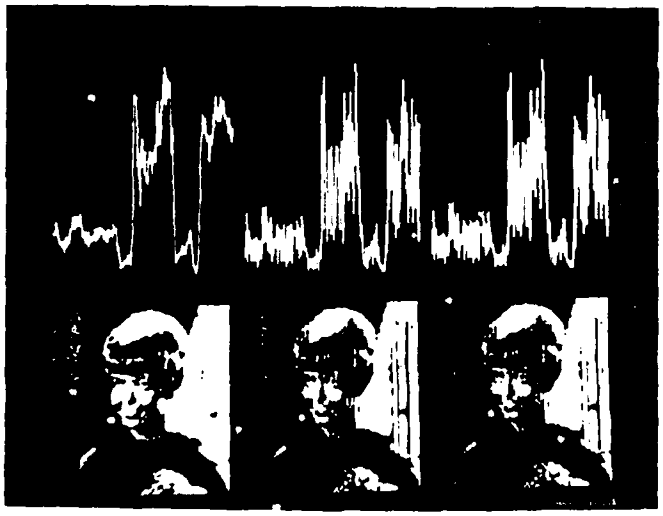

Fig. 2B
Filtering of noisy image with multiplicative noise of uniform distribution (0.2-1.) by using L-method.

The improvement of our method over the L-method is evident by comparing Fig. 2A with Fig. 2B.

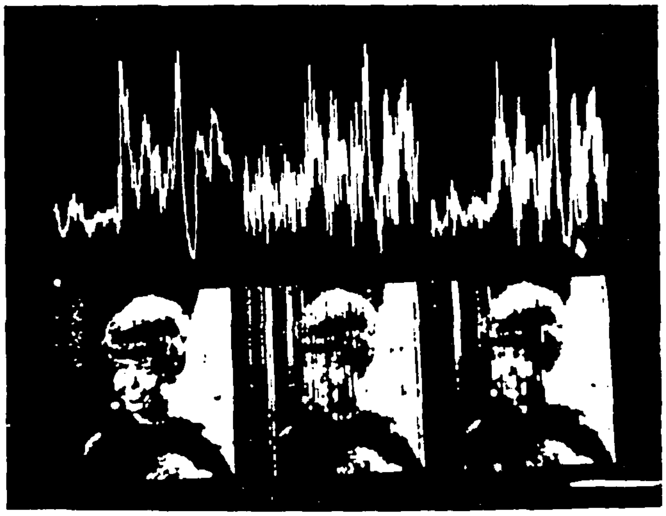

Fig. 3
Filtering of noisy image with mixed additive and multiplicative case. Additive noise, =133.3 and uniform over (-20,20). Multiplicative noise, uniform over (0.2-1.)

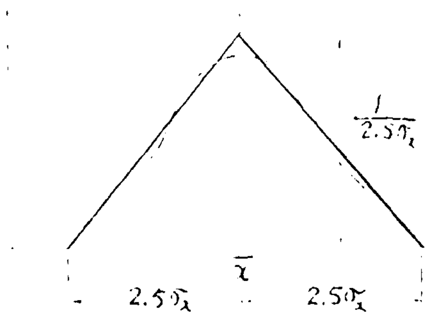

Fig. 4

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An alternative approach is presented for the noise filtering using local statistics, a problem recently considered by Jong-Sen Lee. For the multiplicative noise case improved filtering by the new method is demonstrated by experiemntal results.

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